



Agzamxo'djayeva M.SH

Mavzu: Sonning logarifimi. Asosiy logarifmik ayniyat. Bir asosdan boshqa asosga o'tish formulasi.

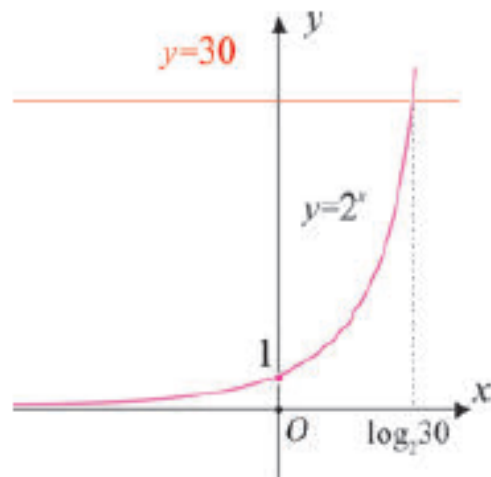
Logarifm haqida tushuncha

$2^x=32$ tenglamaning ildizi $x=5$, ammo $2^x=30$ tenglamaning ildizi qanday topiladi? Bu kabi tenglamalarni yechish uchun sonning logarifmi tushunchasi kiritiladi. $2^x=30$ tenglama yagona ildizga ega. Uni 32- rasmdan ko'rish mumkin.

Bu ildiz 30 sonining 2 asosga ko'ra logarifmi deyiladi va $\log_2 30$ kabi belgilanadi. Demak, $2^x=30$ tenglamaning ildizi $x=\log_2 30$ sonidir.

Ushbu ta'rifni kiritamiz:

b musbat sonning a asosga ko'ra logarifmi deb, b sonni hosil qilish uchun asos a ni ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi va $\log_a b$ kabi belgilanadi. Asos $a>0$ va $a\neq 1$ shartni qanoatlantirishi kerak.



32- rasm.

Masalan, $\log_3 9=2$, chunki $9=3^2$. Shuningdek, $\log_2 \frac{1}{8} = -3$; $\log_5 5=1$; $\log_7 1=0$.

1- misol. Hisoblang: $\log_3 81$.

△ $3^4=81$ bo'lgani uchun logarifmning ta'rifiga ko'ra $\log_3 81=4$. ▲



TIAME

$a > 0, a \neq 1$ bo'lsin. N sonining a asos bo'yicha **logarifmi** deb, N sonini hosil qilish uchun a sonini ko'tarish kerak bo'lgan daraja ko'rsatkichiga aytiladi va $\log_a N$ bilan belgilanadi.

Ta'rifga ko'ra, $a^x = N$ ($a > 0, a \neq 1$) tenglamaning x yechimi $x = \log_a N$ sonidan iborat. Ifodaning logarifmini topish amali shu ifodani *logarifmlash*, berilgan logarifmiga ko'ra shu ifodaning o'zini topish esa *potensirlash* deyiladi.

$x = \log_a N$ ifoda potencirlansa, qaytadan $N = a^x$ hosil bo'ladi. $a > 0, a \neq 1$ va $N > 0$ bo'lgan holda $a^x = N$ va $\log_a N = x$ tengliklar teng kuchlidir.

Logarifmning xossalari

- asosiy logarifmik ayniyat: agar $a > 0$, $a \neq 1$, $b > 0$ bo'lsa, $a^{\log_a b} = b$ tenglik o'rinlidir;
- agar $a > 0$, $a \neq 1$ bo'lsa, $\log_a 1 = 0$; $\log_a a = 1$;
- agar $a > 0$, $a \neq 1$ va $x > 0$, $y > 0$ bo'lsa, $\log_a (xy) = \log_a x + \log_a y$;
- agar $a > 0$, $a \neq 1$ va $x > 0$, $y > 0$ bo'lsa, $\log_a \frac{x}{y} = \log_a x - \log_a y$;
- agar $a > 0$, $a \neq 1$, $x > 0$ bo'lsa $\log_a x^n = n \cdot \log_a x$;
- yangi asosga (bir asosdan boshqa asosga) o'tish formulasi: agar $a > 0$, $a \neq 1$, $x > 0$, $b > 0$, $b \neq 1$ bo'lsa, $\log_a x = \frac{\log_b x}{\log_b a}$;
- agar $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$ bo'lsa, $\log_a b \cdot \log_b a = 1$.

$\log_{10} x = \lg x$ va $\log_e x = \ln x$ kabi belgilash qabul qilingan ($e=2,718281\dots$).

Bunda $\lg x$ ifoda x ning o'nli logarifmi, $\ln x$ esa x ning natural logarifm deyiladi. $f(x) = \log_a x$ funksiya (bu yerda x – argument, $a > 0$, $a \neq 1$) a asosli logarifmik funksiya deyiladi.



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Logarifmik funksiyaning qolgan xossalarini isbotlashda ushbu *asosiy logarifmik ayniyatdan* ham foydalaniladi:

$$a^{\log_a N} = N \quad (N > 0, a > 0, a \neq 1.) \quad (1)$$

(1) ayniyat $a^x = N$ tenglikka $x = \log_a N$ ni qo'yish bilan hosil qilinadi. O'zgaruvchi qatnashgan $a^{\log_a x} = x$ tenglik x ning $x > 0$ qiymatlaridagina o'rinli bo'ladi. $x \leq 0$ da $a^{\log_a x} = x$ ifoda ham o'z ma'nosini yo'qotadi.



$$1) \log_a 1 = 0, \text{ chunki } a^0 = 1;$$

$$2) \log_a a = 1, \text{ chunki } a^1 = a;$$

$$(c > 0, c \neq 1).$$

$$3) \log_a (NM) = \log_a N + \log_a M.$$

Haqiqatan, $NM = a^{\log_a N} \times a^{\log_a M} = a^{\log_a N + \log_a M}$. Ikkinchi tomondan, $NM = a^{\log_a NM}$. Tengliklarning o'ng qismlari tenglashtirilsa, (3) tenglik hosil bo'ladi.

Agar N va M bir vaqtda manfiy bo'lsa, u holda: $\log_a (NM) = \log_a |N| + \log_a |M|$

$$4) \log_a \frac{N}{M} = \log_a N - \log_a M.$$



$$5) \log_a N = \frac{\log_c N}{\log_c a} \quad (c > 0, c \neq 1).$$

Bu tenglik $N = a^c$ tenglikka $N = c^{\log_c N}$, $a = c^{\log_c a}$, $c = \log_a N$ larni qo'yish va almashtirishlarni bajarish orqali hosil bo'ladi.

$$6) \log_a \frac{1}{N} = -\log_a N$$

$$7) \log_a N^\beta = \beta \log_a N$$

Haqiqatan, $x = \log_a N^b$ va $y = \log_a N$ bo'lsin. Ta'rifga ko'ra $N^b = a^x$ va $N = a^y$ yoki $N^b = a^{by}$. Bulardan $a^x = a^{by}$ yoki $x = by$ va (7) tenglik hosil bo'ladi;

$$8) \log_{a^\beta} N = \frac{1}{\beta} \log_a N \quad \beta - \text{haqiqiy son.}$$



Amaliyotda asosi 10 bo‘lgan (*o‘nli* logarifmlar) va asosi $e = 2,7182818\dots$ ga teng bo‘lgan (*natural* logarifmlar) logarifmlar keng qo‘llaniladi. Ularni mos ravishda $\lg N$ va $\ln N$ ko‘rinishda belgilash qabul qilingan.

1-misol. a) $\lg 1000^{67}$; b) $\ln e^{4,8}$ larni hisoblang.

Ye ch i sh : a) $\lg 1000^{67} = \lg 10^3 \times 67 = \lg 10^{201} = 201 \times \lg 10 = 201 \times 1 = 201$;

b) $\ln e^{4,8} = 4,8 \ln e = 4,8 \times 1 = 4,8$.

2-misol. Jadvalda $\lg 3 = 0,4771$ ekanligi berilgan. a) $\lg 270$ ni; b) 3^{1000} ni toping.

Ye ch i sh : a) $\lg 270 = \lg 3^3 \times 10 = 3 \lg 3 + \lg 10 = 3 \times 0,4771 + 1 = 2,4313$.

b) $3^{1000} = x$ deb, bu tenglikni logarifmlasak, $\lg x = 1000 \lg 3 \approx 477,1$ yoki bundan $x \approx 10^{477,1}$ hosil bo‘ladi.

Demak, $3^{1000} = 477,1 \approx 1\,000\dots 0$

5- misol. Hisoblang: $A = 4^{\log_8 125} + 27^{\frac{1}{3} - \frac{1}{2} \log_3 4}$.

△ Logarifmning xossalaridan foydalanamiz: $\frac{1}{2} \log_3 4 = \log_3 2$;

$$\log_8 125 = \frac{\log_2 125}{\log_2 8} = \frac{3 \log_2 5}{3} = \log_2 5; \quad 4^{\log_8 125} = 4^{\log_2 5} = 2^{2 \log_2 5} = 2^{\log_2 25} = 25.$$

Shuningdek, $27^{\frac{1}{3} - \frac{1}{2} \log_3 4} = 27^{\frac{1}{3} - \log_3 2} = 27^{\frac{1}{3}} \cdot 27^{-\log_3 2} =$

$$= 3 \cdot 3^{-3 \log_3 2} = 3 \cdot 3^{\log_3 \frac{1}{8}} = 3 \cdot \frac{1}{8} = \frac{3}{8}. \quad \text{Demak, } A = 25 + \frac{3}{8} = 25 \frac{3}{8}. \quad \blacktriangle$$

6- misol. Hisoblang: $\frac{\lg 54 + \lg \frac{1}{2}}{\lg 72 - \lg 8}$.

△ Logarifmning xossalariidan foydalanamiz:

$$\lg 54 + \lg \frac{1}{2} = \lg\left(54 \cdot \frac{1}{2}\right) = \lg 27 = \lg 3^3 = 3 \lg 3,$$

$$\lg 72 - \lg 8 = \lg \frac{72}{8} = \lg 9 = \lg 3^2 = 2 \lg 3.$$

U holda: $\frac{\lg 54 + \lg \frac{1}{2}}{\lg 72 - \lg 8} = \frac{3 \lg 3}{2 \lg 3} = \frac{3}{2}$. Javob: $\frac{3}{2}$. ▲