IHT ACADEMIC LYCEUM

TIIAME

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Mavzu:Logarifmik tengsizliklar

$\log_a x < b$, $\log_a x > b$, $\log_a x \leq b$, $\log_a x \geq b$ ko‘rinishdagi (bu yerda $a > 0$, $a \neq 1$) tengsizliklar eng sodda ***logarifmik tengsizliklardir.***

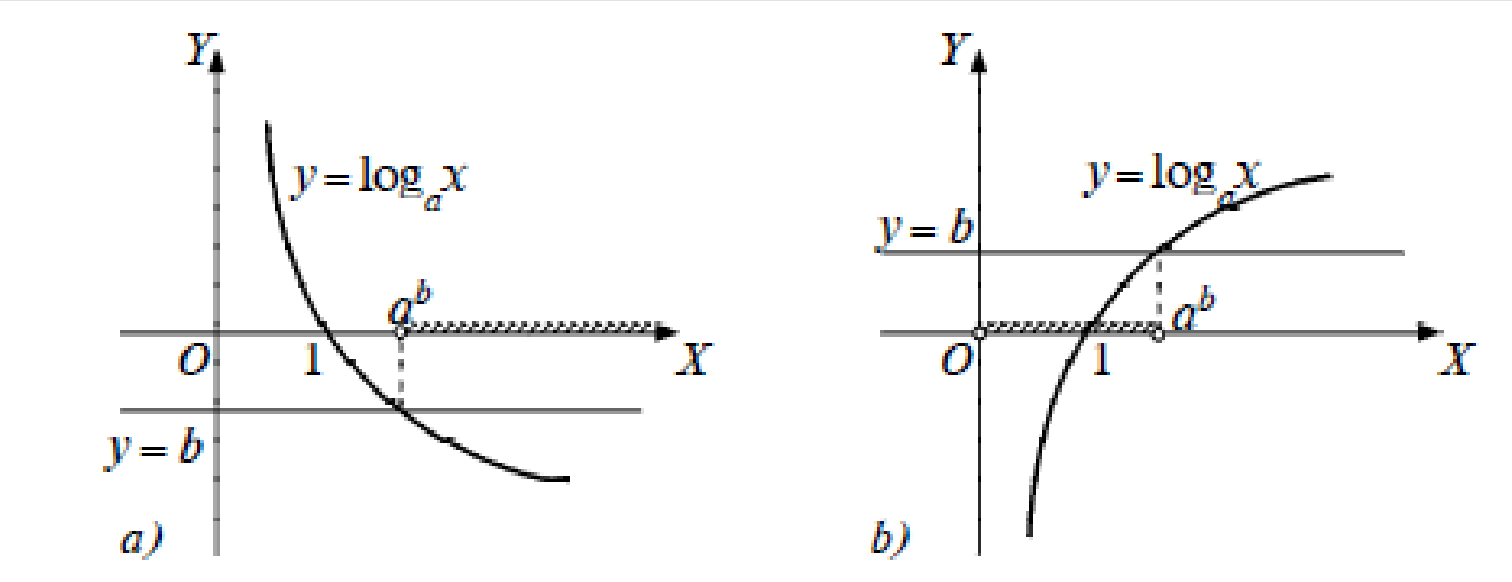
Ularni yechishda $\boldsymbol{y = \log_a x}$ funksiyaning monotonligidan foydalaniladi.

$\boldsymbol{\log_a x < b}$ logarifmik tengsizlikni qaraymiz.

Agar $0 < a < 1$ bo‘lsa, bu tengsizlikning barcha yechimlari to‘plami $(a^b; +\infty)$ oraliqdan iborat bo‘ladi.

Agar $a > 1$ bo‘lsa, qaralayotgan tengsizlikning barcha yechimlari to‘plami $(0; a^b)$ oraliqdan iborat bo‘ladi.

$\log_a x > b$, $\log_a x \leq b$, $\log_a x \geq b$ tengsizliklar ham shunga o‘xshash yechiladi.


Y
$y = \log_a x$
a^b
O
1
X
$y = b$
a)
Y
$y = \log_a x$
$y = b$
a^b
O
1
X
b)

1- m i s o l . a) $\log_3 x < 9$; b) $\log_{\frac{1}{3}} x < 9$ tengsizliklarni yechamiz.

Yechish . a) $\log_3 x = 9$ tenglamaning $x = 3^9$ ildizini topamiz. Asos $a = 3 > 1$, $b = 9$.

Yechim: $(0; 3^9)$ yoki $0 < x < 3^9$;

b) $a = \frac{1}{3} \in (0; 1)$ bo‘lgani uchun yechim $(3^{-9}; +\infty)$ oraliqdan iborat.

Teorema. ***Agar*** $0 < a < 1$ ***bo‘lsa,*** $\log_a f(x) > \log_a g(x)$ ***tengsizlik***

$0 < f(x) < g(x)$ ***qo‘sh tengsizlikka,***

$a > 1$ ***bo‘lsa,*** $f(x) > g(x) > 0$ ***qo‘sh tengsizlikka teng kuchlidir.***

Bu teoremaning isboti logarifmik funksiyaning monotonligidan kelib chiqadi.

13- misol. Tengsizlikni yeching: $\log_{\frac{1}{2}}(3-x) > -3$.

△ $3-x>0$ bo'lishi kerak, $-3=\log_{\frac{1}{2}} 8$ ekanidan $\log_{\frac{1}{2}}(3-x) > \log_{\frac{1}{2}} 8$. Asos $a=\frac{1}{2}<1$ bo'lgani uchun logarifmik funksiya kamayuvchi, demak, $3-x<8$ va $0<3-x<8$. Bundan $-3<-x<5$ yoki $-5<x<3$ tengsizliklarga kelamiz.

Javob: $x\in(-5;\ 3)$. ▲

14- misol. Tengsizlikni yeching: $\lg(x+1) < \lg(2x-3)$.

△ Logarifmik funksiyaning xossalaridan quyidagi tengsizliklar sistemasini olamiz:

$$\begin{cases} x+1<2x-3, \\ x+1>0, \\ 2x-3>0 \end{cases} \quad \text{yoki} \quad \begin{cases} x>4, \\ x>-1, \\ x>\frac{3}{2}. \end{cases}$$

Bu sistemaning yechimi $(4;\ +\infty)$ oraliqdan iborat. *Javob*: $x\in(4;\ +\infty)$. ▲

15- misol. Tengsizlikni yeching: $\log_{\frac{1}{2}}^{2} x - 9 \leq 0$.

△ Logarifmik funksiya ta'rifiga ko'ra, $x > 0$ bo'lishi kerak. $t = \log_{\frac{1}{2}} x$ belgilash kiritamiz. U holda $t^2 - 9 \leq 0$ tengsizlikni hosil qilamiz. Buni yechib $-3 \leq t \leq 3$, ya'ni $-3 \leq \log_{\frac{1}{2}} x \leq 3$ tengsizliklarga kelamiz. $-3 = \log_{\frac{1}{2}} 8$; $3 = \log_{\frac{1}{2}} \frac{1}{8}$ ekanidan $\log_{\frac{1}{2}} 8 \leq \log_{\frac{1}{2}} x \leq \log_{\frac{1}{2}} \frac{1}{8}$. Asos $a = \frac{1}{2} < 1$ bo'lgani uchun $y = \log_{\frac{1}{2}} x$ funksiya kamayuvchi, demak, $\frac{1}{8} \leq x \leq 8$ bo'lishi kerak. *Javob:* $x \in [\frac{1}{8}; 8]$. ▲