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I kurs. GEOMETRIYA

14 -15- Mavzu: Uchburchaklarning o`xshashligi

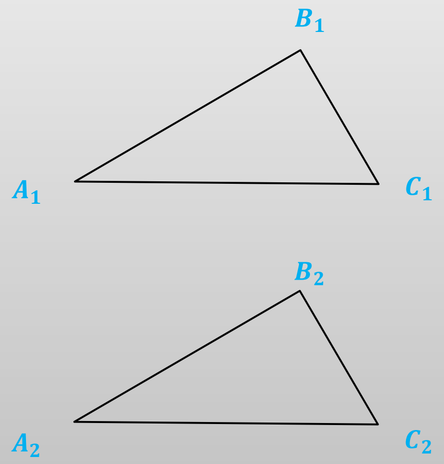
Agar ikkita A1B1C1 va A2B2C2 uchburchak berilgan bo`lib :

1) ularning mos tomonlari o`zaro proporsional, ya‘ni

$$\frac{A_1B_1}{A_2B_2} = \frac{B_1C_1}{B_2C_2} = \frac{A_1C_1}{A_2C_2}$$

2) ularning mos burchaklari o`zaro teng, ya‘ni ∠A1 = ∠A2.

∠B1 = ∠B2, ∠C1 = ∠C2 bo`lsa, bu uchburchaklar o`xshash deyiladi.



O`xshash uchburchaklar mos tomonlarining nisbati bu uchburchaklarning o`xshashlik koeffitsiyenti deb ataladi:

$$\frac{A_1B_1}{A_2B_2} = k$$

1 - T E O R E M A . AGAR BURCHAKNING TOMONLARI PARALLEL TO`G`R CHIZIQLAR BILAN KESILSA, HOSIL QILINGAN UCHBURCHAKLAR O`XSHASH BO_LADI.

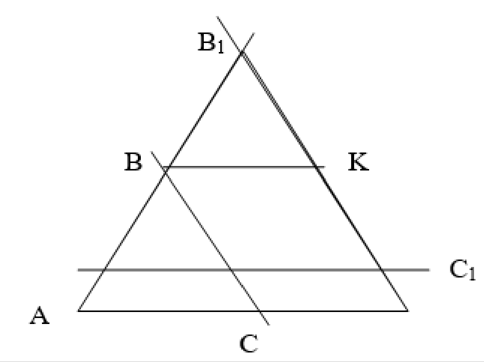
I s b o t i . Bizga ∠BAC berilgan bo_lib, uning tomonlari o`zaro parallel BC va $B_1C_1$ to`g`ri chiziqlar bilan kesilgan , ya'ni BC $B_1C_1$ bo`lsin. Buning natijasida hosil qilingan ABC va $A_1B_1C_1$ ning o`xshashligini isbotlaymiz.

Ularda ∠A—umumiy va o`zaro parallel BC va $B_1C_1$ to`g`ri chiziqlar va $BB_1$ kesuvchi hosil qilgan ∠ABC hamda $∠AB_1C_1$mos burchaklar sifatida bir-biriga tengdir, $∠ABC = ∠A B_1C_1$. Bundan esa uchburchaklarning uchinchi burchaklari ham o`zaro tengligi kelib chiqadi $∠ACB = ∠A B_1C_1$.

Endi uchburchaklarning mos tomonlari proporsionalligini ko`rsatamiz. ∠BAC ning tomonlari o`zaro parallel BC va B_1C_1 to`g`ri chiziqlar bilan kesilganligidan, Fales teoremasiga ko`ra bo`ladi. Bu tenglikning har ikkala tomoniga 1 ni qo`shib, umumiy maxrajga keltiramiz:

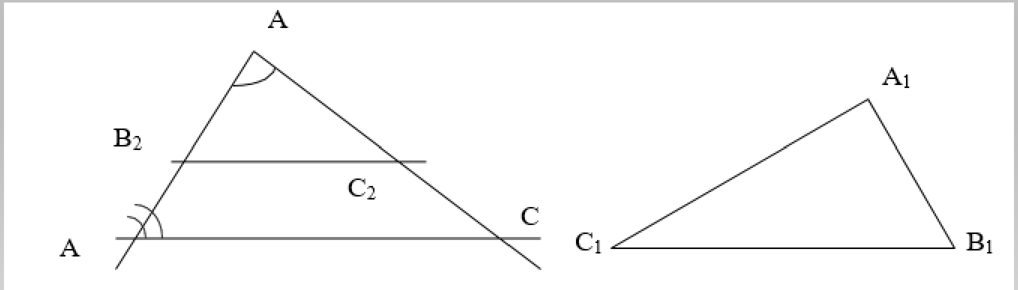
$$\frac{BB_1}{AB} + 1 = \frac{CC_1}{AC} + 1, \quad \frac{BB_1+AB}{AB} = \frac{CC_1+AC}{AC}, \quad \frac{AB_1}{AB} = \frac{AC_1}{AC}$$

Uchinchi tomonlarning ham proporsionalligini ko_rsatamiz. B nuqtadan BK∥AC to`g`ri chiziq o`tkazamiz. BC∥$B_1$ $C_1$, BK∥$CC_1$ bo`lganligidan K C_1 =BC bo`ladi. Shundan qilib, < $AB_1$ $C_1$ burchakning tomonlari o`zaro parallel BK va A C_1 to`g`ri chiziqlar bilan kesilgan, ya'ni BK∥A C_1. Endi, yuqoridagiga o`xshash, Fales teoremasidan foydalanib, $\frac{B_1B_1}{KC_1} = \frac{AB_1}{AB}$ yoki $\frac{B_1C_1}{BC_1} = \frac{AB_1}{AB}$ ekanligini isbotlaymiz. Shunday qilib, $\frac{AB_1}{AB} = \frac{AC_1}{AC} = \frac{B_1C_1}{BC}$ bo`ladi.



2 - T E O R E M A (UCHBURCHAKLAR O`XSHASHLIGINING BIRINCHI ALOMATI). AGAR BIR UCHBURCHAKNING IKKI BURCHAGI IKKINCHI UCHBURCHAKNING, MOS RAVISHDA, IKKI BURCHAGIGA TENG BO`LSA, BU UCHBURCHAKLAR O`XSHASH BO`LADI.

I s b o t i . Teoremaning sharti bo`yicha, ΔABC va ΔA1B1C1 lar uchun A = ∠A1, ∠B = ∠B1 tengliklar bajariladi. Endi ∠Cva∠C1 ning o`zaro tengligi va uchburchaklar mos tomonlarining proporsionalligini ko_rsatish qoldi, xolos. Uchburchak ichki burchaklarining yig`indisi formulasidan, $\angle C = 180° — (\angle A + \angle B) = 80° — (\angle A_1 + \angle B_1) = \angle C_1$. AB tomonda A nuqtadan boshlab AB2 = $A_1$ $B_1$ kesmani ajratamiz va $B_2$ nuqta orqali $B_2C_2$ BC to`g`ri chiziq o`tkazamiz. 1- teoremaga ko`ra, $\Delta AB_2C_2$ ΔABC. Endi $\Delta AB_2C_2 = \Delta A_1B_1C_1$ tenglikni isbotlash qoldi. Yasashga ko`ra $AB_2 = A_1B_1$, shartga ko`ra, $\angle A = \angle A_1$. Modomiki, $B_2C_2 \| BC$ ekan, mos burchaklar sifatida $\angle AB_2C_2 = \angle ABC$ bo`ladi. Lekin shartga ko`ra $\angle B = \angle B_1$ va shuning uchun $\angle AB_2C_2 = \angle A_1B_1C_1$. Uchburchaklar tengligining ikkinchi alomatiga ko`ra $\Delta A_1B_1C_1 = \Delta AB_2C_2$. Modomiki, ΔAB_2C_2 ΔABC ekan, $\Delta A_1B_1C_1$ Δ ABC.



3-T E O R E M A (UCHBURCHAKLAR O`XSHASHLIGINING IKKINCHI ALOMATI). AGAR BIR UCHBURCHAKNING IKKI TOMONI IKKINCHI UCHBURCHAKNING MOS TOMONLARIGA PROPORSIONAL BO`LIB, ULAR ORASIDAGI BURCHAKLAR O`ZARO TENG BO`LSA, UCHBURCHAKLAR O`XSHASH BO`LADI.

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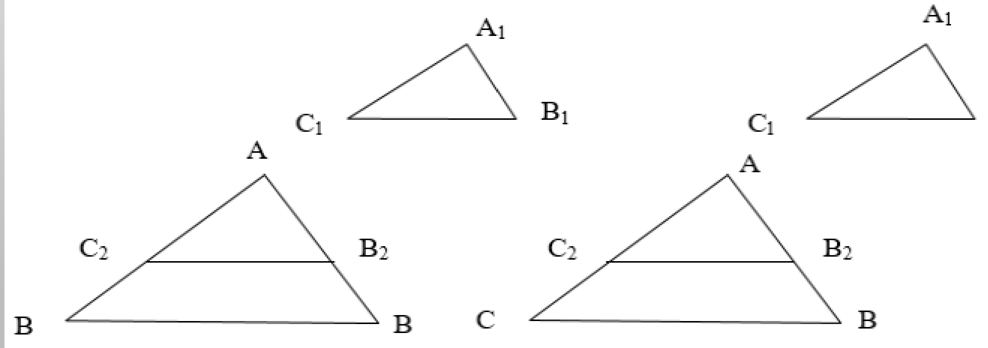
I s b o t i . Teoremaning shartiga ko`r va ∠A = ∠A1 (14-rasm). AB tomonda A uchdan boshlab, AB2 = A1B1 kesma ajratamiz va B2 nuqtadan B2C2||B1C1 to`g`ri chiziqni o`tkazamiz. U vaqtda ΔAB2C2 ∝ ΔABC va bo_ladi. Yasash bo`yicha AB2 = A1B1 bo`lganligidan, hosil qilingan proporsiyalarni (nisbatlarni) taqqoslaymiz. Proporsiyalarning chap tomonlari teng bo_lganligidan ularning o`ng tomonlari ham teng bo`lishi kerak: Bundan AC2 = A1C1 bo_lishi kelib chiqadi. U vaqtda ikki tomoni va ular orasidagi burchagi teng bo_lgan ΔA1B1C1 va ΔAB2C2 o`zaro teng, ya‘ni ΔA1B1C1=ΔAB2C2 bo`ladi. Demak, ΔA1B1C1 ΔABC, teorema isbotlandi.

4 - TEOREMA (UCHBURCHAKLAR O`XSHASHLIGINING UCHINCHI ALOMATI). AGAR BIR UCHBURCHAKNING UCHTA TOMONI IKKINCHI UCHBURCHAKNING UCHTA MOS TOMONLARIGA PROPORSIONAL BO`LSA, BU UCHBURCHAKLAR O`XSHASH BO`LADI.

I s b o t i . Shartga ko`ra , Biz ∠A = ∠A1, ∠B =∠B1, ∠C = ∠C1 bo`lishini isbotlashimiz kerak ΔABC ning A uchidan AB2 = A1B1, AC2= A1C1 kesmalarni ajratamiz. O`xshashlikning yuqorida isbotlangan birinchi alomatiga binoan ΔAB2C2 va ΔABC bo`ladi. AC2 = A1C1 bo`lganligidan, yuqorida yozilgan proporsiyalardan B2C2= B1C1



Bo`lishi kelib chiqadi. U vaqtda uchburchaklar tengligining uchinchi alomati bo`yicha ΔAB2C2 =ΔA1B1C1 bo`ladi. Yasashga ko`ra AB2=A1B2, AC2 = A1C1 hamda isbotlanganiga asosan B2C2 = B1C1 bo_ladi. Modomiki, ΔABC ∞ ΔAB2C2 va ΔAB2C2=ΔA1B1C1 ekan, ABC∞A1B1C1.

5 -O`XSHASH UCHBURCHAKLARNING PERIMETRLARI ULARNING O`XSHASH TOMONLARI KABI NISBATDA BO`LADI.

I s b o t i . ΔABC da P—perimetr, a, b, c — uning tomonlari, ΔA1B1C1 da esa, P1— perimetr, a1,b1,c1 —uning tomonlari bo`lsin va shartga ko`ra, ΔABC A1B1C1. O`xshash uchburchaklarning aniqlanishidan, ularning o`xshash tomonlari proporsional bo`ladi:

Bundan $\frac{a}{a_1} = \frac{b}{b_1}$ tenglikni $\frac{a}{b} = \frac{a_1}{b_1}$ ko`rinishda yozib olamiz. Oxirgi tenglikning har ikki tomoniga 1 ni qo`shamiz:

$\frac{a}{b} + 1 = \frac{a_1}{b_1} + 1$ yoki, $\frac{a+b}{b} = \frac{a_1+b_1}{b_1}, \frac{a+b}{a_1+b_1} = \frac{b}{b_1}$

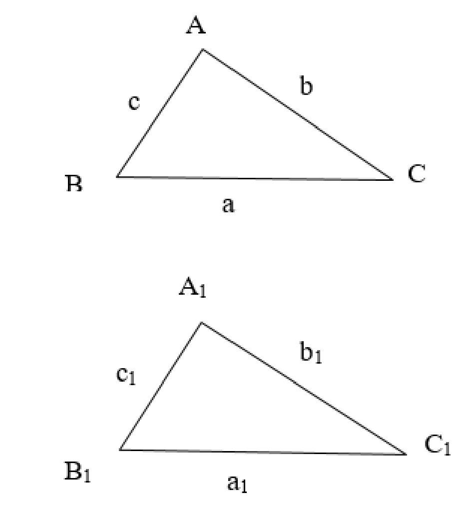
Shunga o`xshash $\frac{b+c}{b_1+c_1}$ $\frac{c}{c_1}$ deb yozish mumkin. U holda

$$\frac{a+b+c}{c} = \frac{a_1+b_1+c_1}{c_1}, \qquad \frac{a+b+c}{a_1+b_1+c_1} = \frac{c}{c_1}$$

munosabatni olamiz. Berilishiga ko`ra

P = a + b + c, P1 = a1 + b1 +c1 bo`lganligidan, talab qilingan munosabatlarni olamiz. Teorema isbotlandi.

6 - T E O R E M A . O`XSHASH UCHBURCHAKLARNING YUZLARI ULARNING O`XSHASH TOMONLARI KVADRATLARI KABI NISBATDA BO`LADI, YA'NI S-ΔABC NING YUZI, S1-ΔA1B1C1 NING YUZI, A VA A1, B VA B1, MOS RAVISHDA, ULARNING O`XSHASH TOMONLARI BO`LSA



I s b o t i . O`xshash ΔABC va Δ $A_1B_1C_1$ da ∠ACB = $A_1B_1C_1$= γ bo`lsin. Unda ularning yuzlari, mos ravishda, S= $ab\frac{1}{2}$ sin γ va S_1= $a_1b_1\frac{1}{2}$ sin γ bo`ladi. S ni S_1 ga

Bo`lamiz: ΔABC∞ Δ $A_1B_1C_1$ bo`lganligidan, yuqorida isbotlanganiga asosan, Shu sababli, va teorema isbotlandi.