# Agzamxodjaeva M.Sh

***Mavzu:***

***Trigonometrik funksiyalarning xossalari va grafigi.***

# Trigonometrik funksiyalarni o`rganishda ishoralar katta rol o`ynaydi

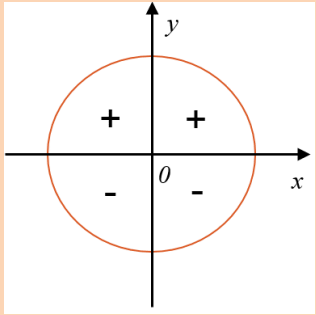
*Choraklarga ajratamiz:*

I chorak $0 < \alpha < \frac{\pi}{2}$

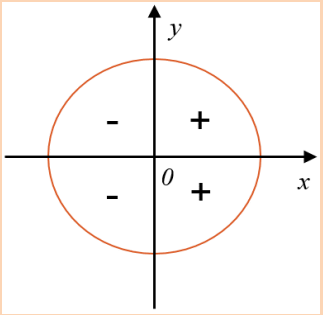
II chorak $\frac{\pi}{2} < \alpha < \pi$

III chorak $\pi < \alpha < \frac{3\pi}{2}$

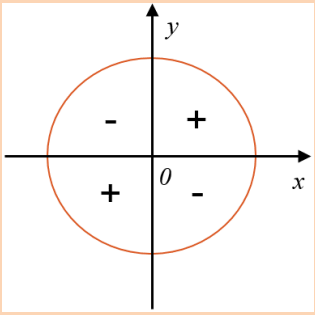
IV chorak $\frac{3\pi}{2} < \alpha < 2\pi$



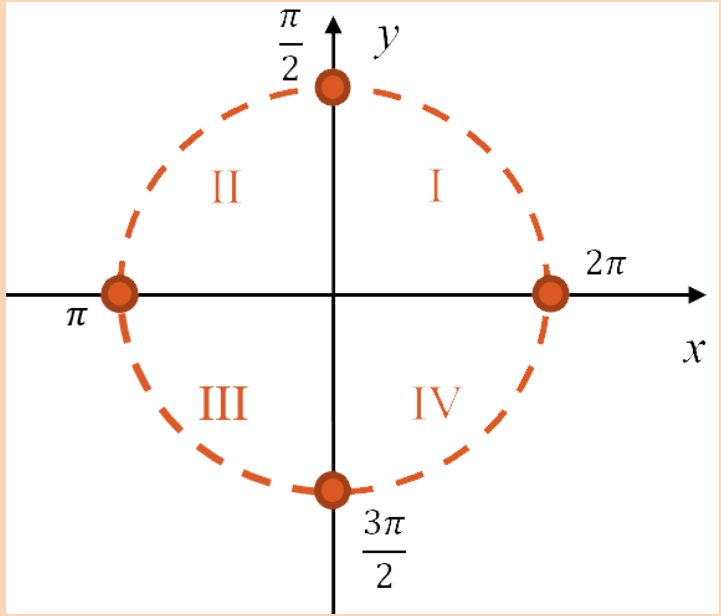

Sinus ishoralari




Kosinus ishoralari




Tangenus va kotangenus ishoralari



# Misol

- Berilgan:

- $\alpha =$

1) $350^o$
2) $-160^o$
3) $2,5$

- Yechim:

- 1) $350^o$ li burchak IV chorakda
- $\sin 350^o < 0$; $\cos 350^o > 0$; $tg 350^o < 0$

- 2) $-160^o$ li burchak II chorakda
- $\sin(-160^o) < 0$; $\cos(-160^o) < 0$; $tg(-160^o) > 0$

- 3) $\frac{\pi}{2} < 2,5 < \pi$ bu II chorak
- $\sin 2,5 > 0$; $\cos 2,5 < 0$; $tg 2,5 < 0$

# Trigonometrik funksiyaning juft va toqligi

- Ta`rif:
- $y = f(x)\ \ x \in D(y); -x \in D(y)$
- $f(-x) = f(x)$ bo`lsa,
- $f$ funksiya juft bo`ladi.

- $f(-x) = -f(x)$ bo`lsa
- $f$ funksiya toq bo`ladi.

- $cos\alpha = cos(-\alpha); sin(-\alpha) = -sin\alpha$
- $tg(-\alpha) = -tg\alpha; ctg(-\alpha) = -ctg(\alpha)$
- $cos\alpha$ – toq; $sin\alpha, tg\alpha, ctg\alpha$ – juft.

- $tg(-\alpha) = \frac{sin(-\alpha)}{cos(-\alpha)} = \frac{-sin(\alpha)}{cos\alpha} = -tg\alpha$

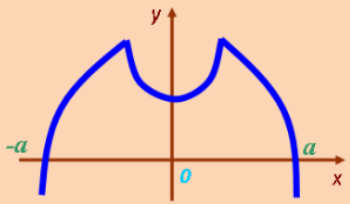
# Trigonometrik funksiyaning davriyligi

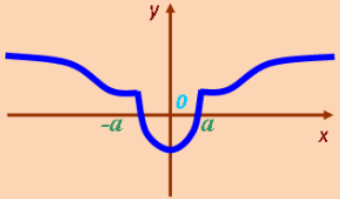
- Ta`rif:
- $y = f(x),\ \exists T \neq 0,\ x; (x - T)\ va\ (x +$

- $sin(\alpha \pm 2\pi) = sin\alpha$
- $cos(\alpha \pm 2\pi) = cos\alpha$
- $tg(\alpha \pm \pi) = tg\alpha$
- $ctg(\alpha \pm \pi) = ctg\alpha$

# Juft va toq funksiyalar

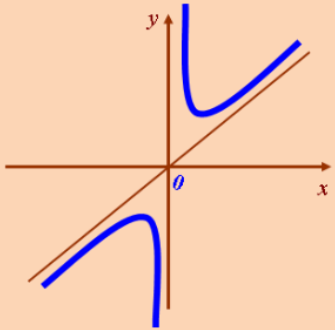
**(1) Juft funksiyalar: $f(-x)=f(x)$**

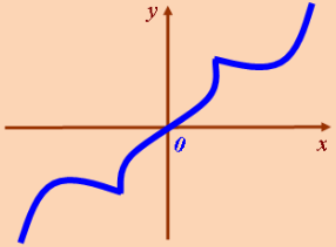



*Ordinatalar o'qi Oy ga nisbatan simmetrik.*

**(2) Toq funksiyalar: $f(-x)=-f(x)$**

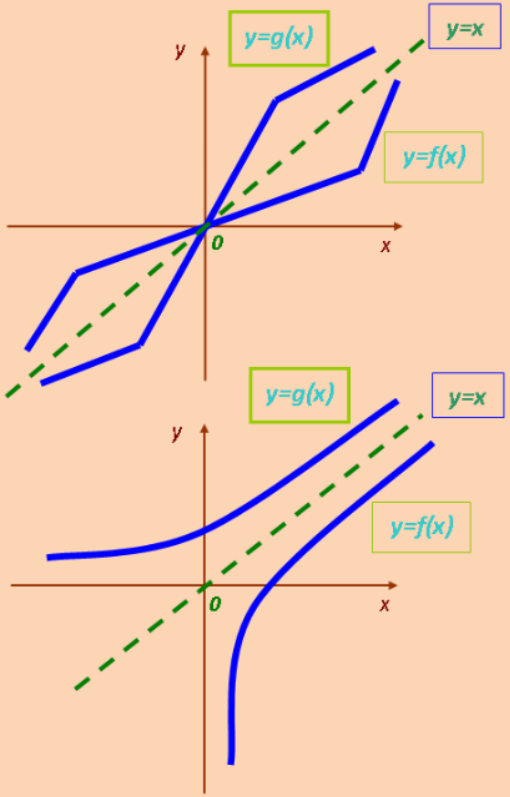



*Koordinatalar boshi 0 nuqtaga nisbatan simmetrik.*

# O'zaro teskari funksiyalar

**y=f(x) va y=g(x) o'zaro teskari funksiyalar**

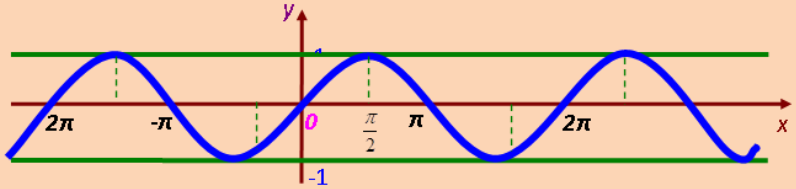



**y=x funksiyaga nisbatan simmetrik**

# Trigonometrik funksiyalar

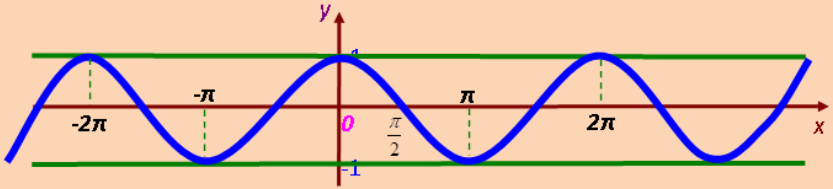
(1) $y=\sin x$, $x \in (-\infty, +\infty)$, $y \in [-1;+1]$

$$\sin(-x)=-\sin x,\ \sin(x+2\pi)=\sin x,\ T=2\pi$$



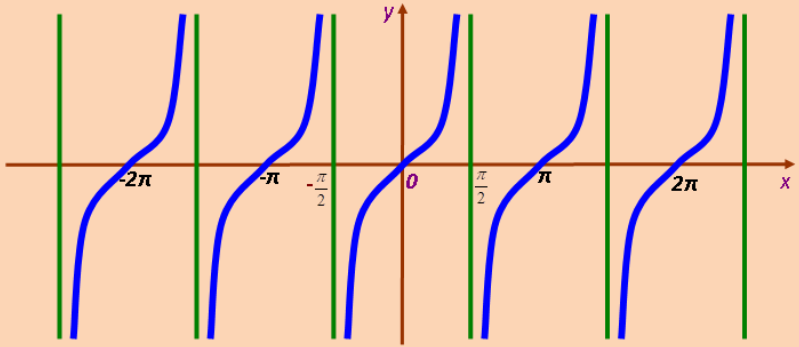

(2) $y=\cos x$, $x \in (-\infty, +\infty)$, $y \in [-1;+1]$

$$\cos(-x)=\cos x,\ \cos(x+2\pi)=\cos x,\ T=2\pi$$



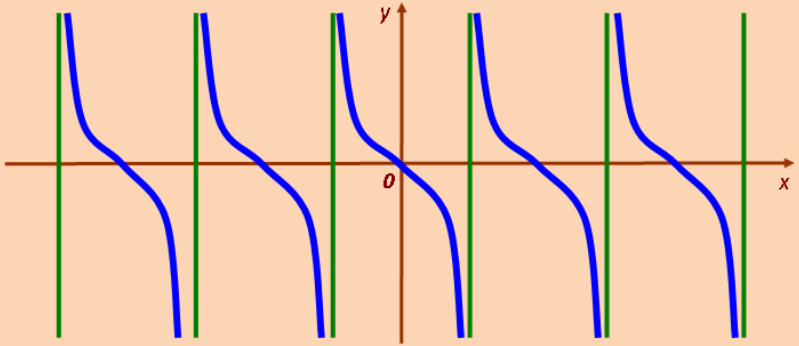
(3) $y=tgx$, $x \in (-\infty, +\infty)$, $y \in (-\infty, +\infty)$

$tg(-x)=-tgx$, $tg(x+\pi)=tgx$, $T=\pi$




(4) $y=ctgx$, $x \in (-\infty, +\infty)$, $y \in (-\infty, +\infty)$

$ctg(-x)=-ctgx$, $ctg(x+\pi)=ctgx$, $T=\pi$

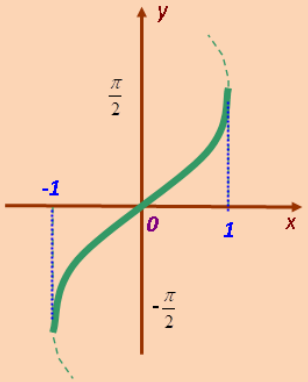


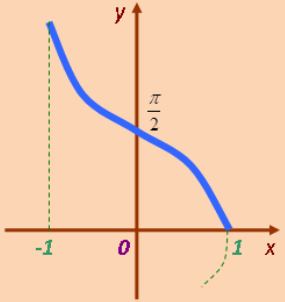
# Teskari trigonometrik funksiyalar

**(1) y=arcsinx**

$x \in [-1;1]$

$y \in [-\frac{\pi}{2};\frac{\pi}{2}]$

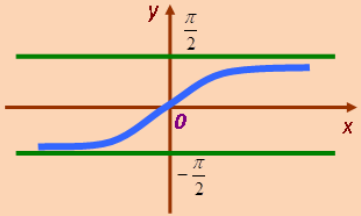



**(2) y=arccosx**

$x \in [-1;1]$

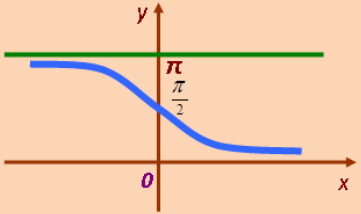
$y \in [0;\pi]$




**(3) y=arctgx**

$x \in (-\infty;+\infty)$

$y \in (-\frac{\pi}{2};\frac{\pi}{2})$



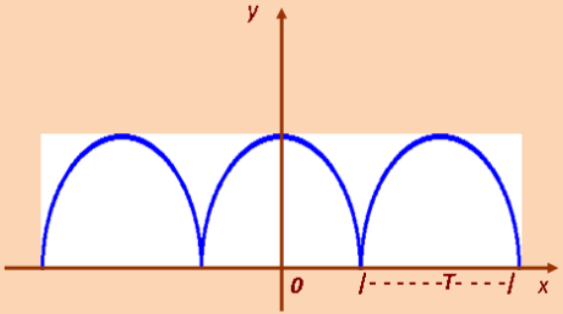

**(4) y=arcctgx**

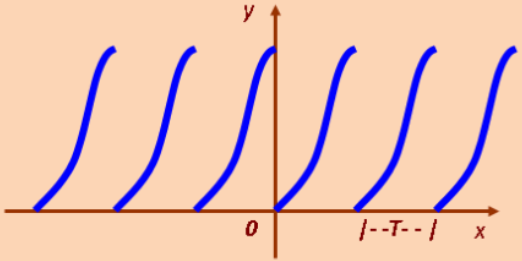
$x \in (-\infty;+\infty)$

$y \in (0;\pi)$

# Davriy funksiyalar

*y=f(x) uchun f(x+T)=f(x); T – davr.*



# $y = sinx$ funksiyaning xossalari

1. $y = sinx$ funkisya x ning barcha qiymatlarida aniqlangan, ya`ni uning aniqlanish sohasi barcha haqiqiy sonlar to`plami R dan iborat.
2. Qiymatlar to`plami $E(x)$ (o`zgarish sohasi) $[-1; 1]$ yopiq oraliq bo`lib, $x = \frac{\pi}{2} + 2\pi k, k \in Z$ qiymatlarida funksiya eng katta qiymatiga ega bo`ladi va bu qiymat 1 ga teng. $X = -\frac{\pi}{2} + 2\pi k, k \in Z$ qiymatlarida esa u eng kichik qiymatiga ega bo`lib, bu qiymat – 1 ga teng.
3. $y = sinx$ funksiya toq funksiya, uning grafigi koordinata boshiga nisbatan simmetrik, ya`ni barcha $x \in R$ da $sin(-x) = sinx$
4. $y = sinx$ funksiya $2\pi k < x < \pi + 2\pi k$ $(k \in Z)$ qiymatlarida musbat, $2\pi + \pi k < x < 2\pi + 2\pi k$ $(k \in Z)$ qiymatlarida esa 0 ga teng.
5. $y = sinx$ funksiya $-\frac{\pi}{2} + 2\pi k < x < \frac{3\pi}{2} + 2\pi k$ qiymatlarida monoton o`sadi, $\frac{\pi}{2} + 2\pi k < x < \frac{3\pi}{2} + 2\pi k$ qiymatlarida monoton kamayadi.
6. $y = sinx$ funksiya $2\pi$ davrli davriy funksiya bo`lib, uning grafigini $[0; 2\pi]$ oraliqda chizish yetarli. Boshqa oraliqlarda grafikni chizish uchun esa grafikni $2\pi, 4\pi, 6\pi$ va hokazo masofaga surish yetarli. Hosil bo`lgan chiziq sinusoida deyiladi.

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# $y = cosx$ funksiyaning xossalari

1. $y = cosx$ funkisya aniqlanish sohasi $D(f)$barcha haqiqiy sonlar to`plami R dan iborat.
2. $y = cosx$ funkisyaning (o`zgarish sohasi) $E(f)$ $[-1; 1]$ yopiq kesmadan iborat bo`lib, $x = 2\pi k, k \in Z$ qiymatlarida funksiya eng katta qiymatiga ega bo`ladi va bu qiymat 1 ga teng. $\mathbf{x} = 2\pi + 2\pi k, k \in Z$ qiymatlarida esa u eng kichik qiymatiga ega bo`ladi.
3. $y = cosx$ funksiya juft funksiya, uning grafigi $Oy$ o`qiga nisbatan simmetrik, ya`ni $\cos(-x) = cosx$
4. $y = cosx$ funksiya $-\frac{\pi}{2} + 2\pi k < x < \frac{\pi}{2} + 2\pi k$ $(k \in Z)$ qiymatlarida musbat, $\frac{\pi}{2} + 2\pi k < x < \frac{3\pi}{2} + 2\pi k$ $(k \in Z)$ qiymatlarida esa manfiy.
5. $y = cosx$ funksiya $2\pi k < x < \pi + 2\pi k$ qiymatlarida monoton kamayuvchi, $\pi + 2\pi k < x < 2\pi + 2\pi k$ qiymatlarida monoton o`sadi.
6. $y = cosx$ funksiya $2\pi$ davrli davriy funksiya bo`lib, uning grafigini $[0; 2\pi]$ oraliqda chizish yetarli. Hosil bo`lgan chiziq kosinusoida deyiladi va uni $y = sinx$ funksiya grafigini chapga $\frac{\pi}{2}$ masofaga qadar surilgan sinusoida deb qarash mumkin.

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# $y = tgx$
# funksiyaning xossalari

1. Aniqlanish sohasi $x \neq \frac{\pi}{2} + \pi n, n \in Z$ qiymatlarida aniqlangan
2. Qiymatlar sohasi $y \in R$, yoki $(-\infty; \infty)$ ya`ni o`zgarish sohasi barcha haqiqiy sonlar to`plami R dan iborat.
3. $y = tg(-x) = -tgx$, toq funksiya.
4. Davriy funkisya, davri $T = \pi$
5. $y = tgx$ funksiya o`zining aniqlanish sohasida monoton o`suvchi.

- Eslatma: $y = tgx$ funksiya $x = \frac{\pi}{2} + \pi k, k \in Z$, qiymatlarida aniqlanmagan, ya`ni uzilishga ega, $x = \pm\frac{\pi}{2}, x = \pm\frac{3\pi}{2}, \ldots$ to`g`ri chiziqlar $y = tgx$ funksiya grafigiga asimptota deyiladi.

# $y = ctgx$
# funksiyaning xosslari

1. Aniqlanish sohasi $x \neq \pi n, n \in Z$ qiymatlarida aniqlangan
2. Qiymatlar sohasi $y \in R$, yoki $(-\infty; \infty)$ ya`ni o`zgarish sohasi barcha haqiqiy sonlar to`plami R dan iborat.
3. $y = ctg(-x) = -ctgx$, toq funksiya.
4. Davriy funkisya, davri $T = \pi$
5. $y = ctgx$ funksiya o`zining aniqlanish sohasida monoton kamayuvchi.

- Eslatma: $y = ctgx$ funksiya $x = \pi k, k \in Z$, qiymatlarida aniqlanmagan, ya`ni uzilishga ega, $x = 0, x = \pm\pi, x = \pm 2\pi$,… to`g`ri chiziqlar $y = ctgx$ funksiya grafigiga asimptota deyiladi. $x \to \pi$ bo`lganda $ctgx \to -\infty$, $x \to 0$ bo`lganda $ctgx \to +\infty$
- $y = ctgx$ funksiyaning grafigini $(0; \pi)$ oraliqda chizish yetarli. Uni $\pi, 2\pi, 3\pi, \dots \pi k$ … masofaga ketma ket ko`chirish yo`li bilan $y = ctgx$ funksiyaning to`la grafigi hosil qilinadi.

# Misollar

1. Funksiyaning davrini toping.

- $y = cos(8x + 1)$ berilgan funksiyaning davrini toppish uchun $8x$ ni $cosx$ ning davriga tenglashtirib hisoblash kifoya. $cosx$ ning davri $2\pi$ ligi bizga ma`lum, bundan $8x = 2\pi \Rightarrow x = \frac{2}{8}\pi \Rightarrow \frac{1}{4}\pi, T = \frac{\pi}{4}$

2. Funksiyaning eng kichik va eng katta qiymatlarini toping;

- $y = 5sin(2x + 3)$, bu funksiyaning qiymatlar sohasi $[-1; 1]$ bo`lganligi uchun funksiyani shu oraliqda olib hisblaymiz. $-1 \leq sin(2x + 3) \leq 1$ va 5 ga ko`paytiramiz. $-5 \leq sin5(2x + 3) \leq 5$. Eng katta qiymati 5 va eng kichik qiymati – 5 dan iborat.

3. Funksiyaning o`zgarish sohasini toping: $y = -5 + 4cosx$

- $-1 \leq cosx \leq 1$ bo`lganligidan, uni 4 ga ko`paytirib, keyin – 5 ni qo`shsak, $-9 \leq -5 + 4cosx \leq -1$. $[-9; -1]$.