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12-Mavzu: Trigonometrik tenglamalarni yechish usullari

Trigonometrik tenglamalarni ko'paytuvchilarga ajratish yo'li bilan tenglamani yechish

Misol. $2\cos x\cos 2x = \cos x$ tenglamani yeching.

Yechish.

$$\cos x(2\cos 2x - 1) = 0 \Leftrightarrow \left[\begin{array}{l}\cos x = 0\\ 2\cos 2x = 1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = \frac{\pi}{2} + \pi n, n \in Z\\ 2x = \pm\frac{\pi}{3} + 2\pi k, x = \pm\frac{\pi}{6} + k\pi, k \in Z\end{array}\right.$$

Javob: $\frac{\pi}{2} + n\pi, \pm\frac{\pi}{6} + \pi k \ (k, n \in Z)$

a) $\sqrt{2}\cos^2 7x - \cos 7x = 0$

b) $2\sin x + tgx = 0$

v) $3\cos x - ctgx = 0$

Kvadrat tenglamaga keltiriladigan tenglamalarni yechish.

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Misol. $3\cos^2 x - 10\cos x + 3 = 0$ tenglamani yeching.

Yechish. $\cos x = t, 3t^2 - 10t + 3 = 0 \; t_1 = \frac{1}{3}, t_2 = 3$

$t_2 = 3$ qiymat shartni qanoatlantirmaydi, chunki $|\cos x| \le 1$

Bundan $\cos x = \frac{1}{3}$; $x = \pm\arccos\left(\frac{1}{3}\right) + 2\pi n, n \in Z$

Javob: $\pm\arccos\left(\frac{1}{3}\right) + 2\pi n, (n \in Z)$

Bir jinsli va unga keltiriladigan tenglamalarni yechish.

$a_0 \sin^n x + a_1 \sin^{n-1} x \cos x + a_2 \sin^{n-2} \cos^2 x + ... + a_{n-1} \sin x \cos^{n-1} x + a_n \cos^n x = 0,$

$a_0, a_1, ..., a_n$ – haqiqiy sonlar,

$\cos x$ nisbatan bir jinsli deyiladi. $\cos \neq 0$ da

$$a_0 tg^n x + a_1 tg^{n-1} x + ... + a_n = 0 \text{ tenglamaga ekvivalentdir.}$$

Misol. Tenglamani yeching

$$6\sin^2 x - \sin x \cos x - \cos^2 x = 3.$$

Yechish.

$$6\sin^2 x - \sin x \cos x - \cos^2 x - 3(\sin^2 x + \cos^2 x) = 0$$

$$3\sin^2 x - \sin x \cos x - 4\cos^2 x = 0$$

$$x = \frac{\pi}{2} + \pi n$$

tenglamani yechimi emas va

$$\cos x \neq 0,$$

$$3tg^2 x - tgx - 4 = 0,$$

$$tgx = -1, x = -\frac{\pi}{4} + \pi n, n \in Z, \qquad \text{va} \cos x \neq 0$$

$$tgx = \frac{4}{3}, x = arctg\left(\frac{4}{3}\right) + \pi k, k \in Z.$$

Javob: $-\frac{\pi}{4} + n\pi, arctg\left(\frac{4}{3}\right) + \pi k, (k, n \in Z)$

Trigonometrik funksiyalar yig’indisini ko’paytmaga keltirish yordamida tenglamani yechish.

$$\sin\alpha \pm \sin\beta = 2\sin\frac{\alpha\pm\beta}{2}\cos\frac{\alpha\mp\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$$

6-misol. $\cos 3x + \sin 2x - \sin 4x = 0$ tenglamani yeching.

Yechish: $\cos 3x + (\sin 2x - \sin 4x) = 0$

$$\cos 3x + (-2\sin x\cos 3x) = 0$$

$$\cos 3x(1 - 2\sin x) = 0$$

$$\cos x = 0$$

$$\sin x = \frac{1}{2}, x = \frac{\pi}{6} + \frac{\pi}{3}n, x = (-1)\frac{\pi}{6} + \pi k, (n, k \in Z).$$

$x = (-1)^k\frac{\pi}{6} + \pi k, (k \in Z)$ yechimlar to’plami to’laligicha

$x = \frac{\pi}{6} + \frac{\pi n}{3}(n \in Z)$ yechimlar to’plamida saqlanadi

Javob: $\frac{\pi}{6} + \frac{\pi}{3}n.(n \in Z)$

Darajani pasaytirish yordamida tenglamani yechish.

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \qquad \sin^2\alpha = \frac{1+\cos 2\alpha}{2}$$

Misol. $\sin^2 x + \sin^2 2x = 1$ tenglamani yeching.

Yechish.

$$\frac{1-\cos 2x}{2} + \frac{1-\cos 4x}{2} = 1 \Rightarrow \cos 2x + \cos 4x = 0 \Rightarrow 2\cos 3x \cos x = 0 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\cos 3x = 0\\ \cos x = 0\end{array}\right. \Rightarrow 3x = \frac{\pi}{2} + \pi n, x = \frac{\pi}{2} + \pi k, \Rightarrow x = \frac{\pi}{6} + \frac{\pi}{3} n, n \in Z, k \in Z$$

Javob: $x = \dfrac{\pi}{6} + \dfrac{\pi n}{3}, (n \in Z)$

Ikkilangan va uchlangan argumentlar formulasini qo'llash yordamida tenglamani yechish.

a) $\sin 2x = \sqrt{2}\cos x$

b) $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x.$

v) $\sin x - 2\cos 2x = 1$

g) $\cos 2x - 3\cos x = 4\cos^2\frac{x}{2}$

a) $\sin\frac{2}{3}x = \frac{1}{2}$ tenglamani yeching.

Yechish. Bu tenglamani yechimi

$$\frac{2}{3}x = (-1)^n \arcsin\frac{1}{2} + n\pi, \frac{2}{3}x = (-1)^n\frac{\pi}{6} + \pi n, x = (-1)^n\frac{\pi}{4} + \frac{2}{3}n\pi, n \in z$$

iborat

$$Ж: x = (-1)^n\frac{\pi}{4} + \frac{3}{2}n\pi, n \in z$$

b) $\sin\frac{3}{4}x = \frac{\sqrt{3}}{2};$

Ikkilangan va uchlangan argumentlar formulasini qo'llash yordamida tenglamani yechish.

v) $\sin\frac{4\pi}{x^2}=1$

$x=\pm\frac{2\sqrt{2}}{\sqrt{4n+1}}, n\in N_0$

g) $\sin\sqrt{\frac{\pi}{x}}=0, \left(x=\frac{1}{k^2\pi}, k\in N\right)$

2) $\cos x=a$ ko'rinishdagi tenglama

a) $\cos(1-2x)=-\frac{\sqrt{2}}{2}$

$\cos(2x-1)=-\frac{\sqrt{2}}{2}, 2x-1=\pm\arccos\left(-\frac{\sqrt{2}}{2}\right)+2n\pi,$

$2x-1=\pm\left(\pi-\arccos\frac{\sqrt{2}}{2}\right)+2n\pi, 2x=1\pm\left(\pi-\frac{\pi}{4}\right)+2n\pi,$

$2x=1\pm\frac{3\pi}{4}+2n\pi, x=\frac{1}{2}\pm\frac{3\pi}{8}+n\pi, n\in z.$

$Ж: x=\frac{1}{2}\pm\frac{3\pi}{8}+n\pi, n\in z$

Ikkilangan va uchlangan argumentlar formulasini qo'llash yordamida tenglamani yechish.

$tgx = a,\ a \in R$ ko'rinishdagi tenglama

$$tg\frac{2}{3x} = -1$$

$$tg\frac{2}{3x} = -1, \frac{2}{3x} = arctg(-1) + n\pi, \frac{2}{3x} = -arctg1 + n\pi,$$

$$\frac{2}{3x} = -\frac{\pi}{4} + n\pi, \frac{2}{3x} = (4n-1)\frac{\pi}{4}, \frac{1}{x} = (4n-1)\frac{3\pi}{8},$$

$$x = \frac{8}{(4n-1)3\pi}, n \in z$$

$ctgx = a,\ a \in R$ ko'rinishidagi tenglama

$$ctgx = -\frac{\sqrt{3}}{3}, 3x = arcctg\left(-\frac{\sqrt{3}}{3}\right) + n\pi, 3x = \pi - arcctg\frac{\sqrt{3}}{3} + n\pi,$$

$$3x = \pi - \frac{\pi}{3} + n\pi, 3x = \frac{2}{3}\pi + n\pi, 3x = (3n+2)\cdot\frac{\pi}{3}, x = (3n+2)\cdot\frac{\pi}{9}, n \in Z$$

$$Ж: x = (3n+2)\cdot\frac{\pi}{9}, n \in Z$$