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17 - Mavzu: KOMPLEKS SONLAR va ULAR USTIDA AMALLAR

KOMPLEKS SONLAR TA'RIFI

Kompleks son deb

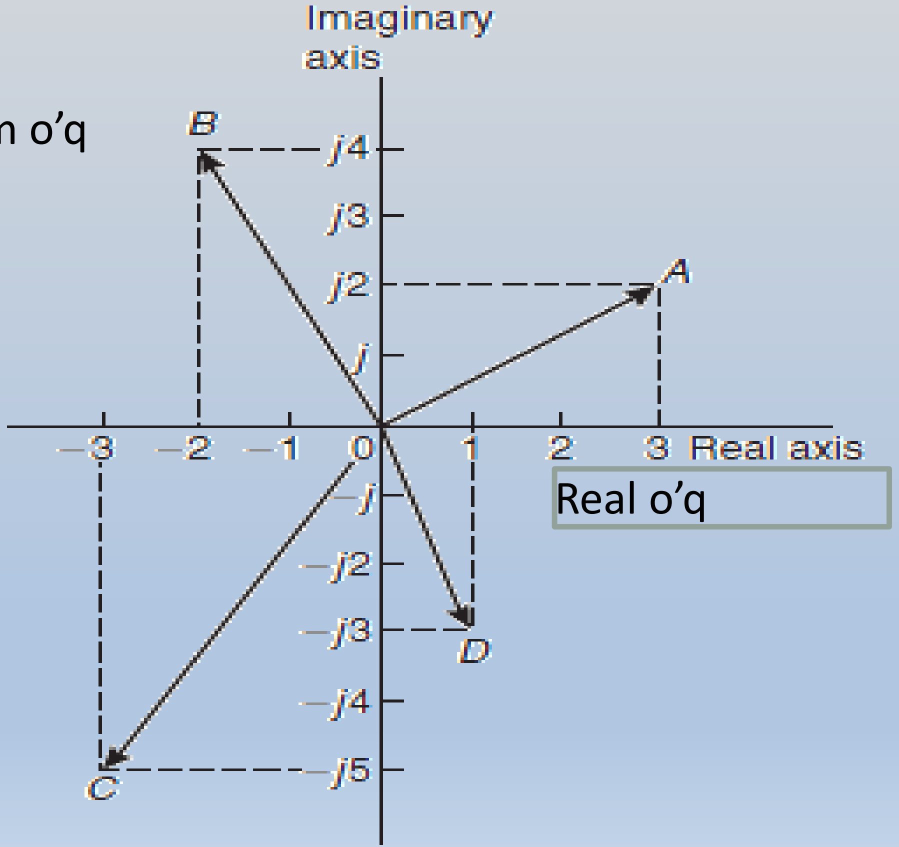
$$z = a + ib \quad (1)$$

ifodaga aytiladi, bu erda *a* va *b* haqiqiy sonlar, *i* - mavhum birlik, ushbu tengliklar bilan aniqlanadi:

$$i = \sqrt{-1} \quad \text{yoki} \quad i^2 = -1 \quad (2)$$

a- kompleks son z ning haqiqiy qismi, *ib* - mavhum qismi deyiladi. Ular bunday belgilanadi: *a=Re z, b=Imz*. Agar *a=0* bo'lsa, *0+ib=ib* sof mavhum son deyiladi; *b=0* agar bo'lsa, haqiqiy son hosil bo'ladi: *a+I*0=a*. Faqat mavhum qismining ishorasi bilan farq qiladigan ikki kompleks son: *z=a+ib* va *z=a-ib* bir-biriga qo'shma deyiladi.

Mavhum o'q



$$Z=3+2i;$$

$$Z=1-3i;$$

$$Z=-2+4i;$$

$$Z=-3+5i$$

KOMPLEKS SONNING TRIGONOMETRIK SHAKLI.

- Koordinatalar boshini qutb, Ox o'qining musbat yo'nalishini qutb o'qi deb olib, *A(a,b)* nuqtaning qutb koordinatalarini φ va $r(r \geq 0)$ bilan belgilaymiz. Unda ushbu tengliklarni yozish mumkin:
- $a = r\cos\varphi$ $b = r\sin\varphi$
- demak, kompleks son z ni bunday tasvirlash mumkin:
- *a+ib*=rcos φ+ *irsin* φ

yoki

$$z = r(\cos\varphi + i\sin\varphi) \quad (3)$$

- Bu tenglikning o'ng tomonidagi ifodada *z=a+ib* kompleks son yozuvining trigonometrik shakli deb ataladi.

KOMPLEKS SONLARNI QO'SHISH.

- Ikki kompleks son $z_1=a_1+ib_1$ va $z_2=a_2+ib_2$ ning yig'indisi deb ushbu
- $z_1+z_2=(a_1+ib_1)+(a_2+ib_2)=(a_1+a_2)+i(b_1+b_2)$
- (1) tenglik bilan aniqlangan kompleks songa aytiladi.
-
  - formuladan vektorlar bilan tasvirlangan kompleks sonlarni

qo'shish-vektorlarni qo'shish qoidasiga muvofiq bajarilishi kelib chiqadi.

KOMPLEKS SONLARNI AYIRISH

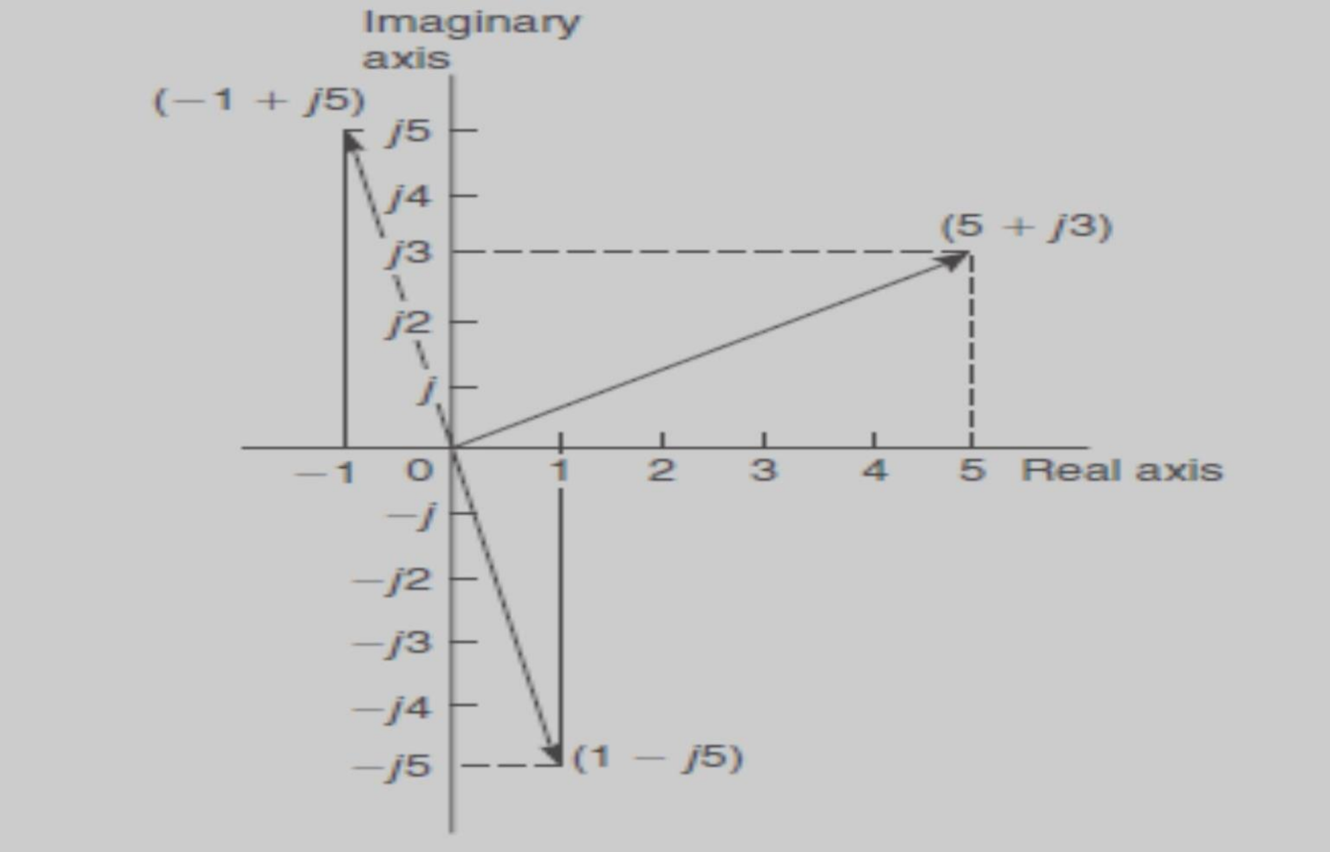
- Ikki $z_1=a_1+ib_1$ **va** $z_2=a_2+ib_2$ kompleks sonlarni ayirmasi deb shunday kompleks songa aytiladiki, unga z_2 kompleks sonni qo'shganda z_1 kompleks son hosil bo'ladi:
- $z_1 - z_2=(a_1+ib_1)- (a_2+ib_2)=(a_1 - a_2)+i(b_1 - b_2)$
  Ikki kompleks son ayirmasining moduli shu sonlarni kompleks
- o'zgaruvchilar tekisligida tasvirlovchi nuqtalar orasidagi masofaga teng:
-

$$|z_1 - z_2| = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$$

MISOL

$Z_1 = 2 + j4$; $Z_2 = 3 - j$ sonlar berilgan.Quyidagi amallarni bajaring

$$\text{(a) } Z_1 + Z_2 = (2 + j4) + (3 - j)$$
$$= (2 + 3) + j(4 - 1) = 5 + j3$$
$$\text{(b) } Z_1 - Z_2 = (2 + j4) - (3 - j)$$
$$= (2 - 3) + j(4 - (-1)) = -1 + j5$$
$$\text{(c) } Z_2 - Z_1 = (3 - j) - (2 + j4)$$
$$= (3 - 2) + j(-1 - 4) = 1 - j5$$


Imaginary
axis
(−1 + j5)
j5
j4
(5 + j3)
j3
j2
j
−1
0
1
2
3
4
5
Real axis
−j
−j2
−j3
−j4
−j5
(1 − j5)

KOMPLEKS SONLARNI KO'PAYTIRISH.

- $z_1=a_1+ib_1$ va $z_2=a_2+ib_2$ kompleks sonlar ko'paytmasi deb, ularni ikki xadlar singari algebra qoidasiga muvofiq, lekin

$i^2=-1$, , $i^3=-i$, $i^4=(-i)i=-i^2=1$, $i^5=i$ va hokazo.

umuman k butun bo'lganda:

$i^{4k}=-1$, ,$i^{4k+1}=i$, $i^{4k+2}=-1$, $i^{4k+3}=-i$

ekanligini e'tiborga olib ko'paytirganda hosil bo'lgan kompleks songa aytiladi.

- Shu qoidaga asosan quyidagi ko'paytmani hosil qilamiz:
- yoki $z_1z_2=(a_1+ib_1)(a_2+ib_2)=a_1a_2+ib_1a_2+ia_1b_2+i^2b_1b_2$
- $z_1z_2=(a_1a_2-b_1b_2)+i(b_1a_2+a_1b_2)$

MISOL. $Z=(3+J2)$ VA $Y=(4-J5)$ KOMPLEKS SONLAR KO'PAYTMASINI TOPING.

$$
\begin{aligned}
(3+j2)(4-j5) &= 12 - j15 + j8 - j^2 10 \\
&= (12-(-10)) + j(-15+8) \\
&= 22 - j7
\end{aligned}
$$

KOMPLEKS SONLARNI BO'LISH.

- Bo'linmani topish uchun surat va maxrajini maxrajga qo'shma bo'lagan songa ko'paytiramiz:

- $$\frac{a_1+ib_1}{a_2+ib_2} = \frac{a_1+ib_1}{a_2+ib_2} * \frac{a_1-ib_1}{a_2-ib_2} = \frac{a_1a_2+b_1b_2}{a_2{}^2+b_2{}^2} +$$

$$+ i\frac{a_2b_1-a_1b_2}{a_2{}^2+b_2{}^2}$$

MISOL. *Z=2-J5 VA Y=3+J4* BERILGAN, BO'LISH AMALINI BAJARING.

$$\frac{2-j5}{3+j4}=\frac{2-j5}{3+j4}\times\frac{(3-j4)}{(3-j4)}$$

$$=\frac{6-j8-j15+j^2 20}{3^2+4^2}$$

$$=\frac{-14-j23}{25}=\frac{-14}{25}-j\frac{23}{25}$$

KOMPLEKS SONLAR TRIGONOMETRIK SHAKLDA

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1),$$

$$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2)$$

- berilgan bo'lsa, ushbuni hosil qilamiz:

$$\frac{z_1}{z_2} = \frac{r_1(\cos\varphi_1 + i\sin\varphi_1)}{r_2(\cos\varphi_2 - i\sin\varphi_2)} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)] \qquad (5)$$

- Bu tenglikni tekshirish uchun bo'luvchini bo'linmaga ko'paytirish kifoya:
- $r_2(\cos\varphi_2 + i\sin\varphi_2)\frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)] =$
- $= r_2\frac{r_1}{r_2}[\cos(\varphi_2 + \varphi_1 - \varphi_2) + i\sin(\varphi_2 + \varphi_1 - \varphi_2)] = r_1(\cos\varphi_1 + i\sin\varphi_1).$

KOMPLEKS SONNI DARAJAGA KO`TARISH.

- Bundan oldingi paragrafdagi (3) formuladan, agar *n* butun musbat son bo'lsa, ushbu formula kelib chiqadi:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i\sin n\varphi). \qquad (1)$$

- Bu Muavr formulasi deb ataladi. Bundan ko'rinadiki, kompleks sonni butun musbat darajaga ko'tarishda modul shu darajaga ko'tariladi, argument esa daraja ko'rsatkichiga ko'paytiriladi.
- Endi Muavr formulasining yana bir tadbiqini qaraymiz.
- Bu formulada r=1 deb faraz qilib,

$$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$$

- tenglikni hosil qilamiz. Chap tomonni Nyuton binomi formulasi bo'yicha yoyib, haqiqiy va mavhum qismlarini tenglab $\sin, n\varphi$ va $\cos n\varphi$ ni $\sin\varphi$ va $\cos\varphi$ning darajalari orqali ifoda qila olamiz.

MUSTAQIL YECHISH UCHUN MISOLLAR

TIIAME

1 $z_1 = 1 + i\sqrt{3}$ $z_2 = 1 - i\sqrt{3}$

$z_1 \cdot z_2 = ?$ $z_1 + z_2 = ?$ $z_1 - z_2 = ?$ $\frac{z_1}{z_2} = ?$

2 $z = \frac{1}{(1 - i\sqrt{3})^6}$

4 $(-1)^{\sqrt{3}}$

3 $z = (1 + i\sqrt{3})^{15}$